Sample Exam I

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I) Problem 1 : (16 points)

Verify Stokes theorem for the vector field and the surface S the portion of the paraboloid satisfying with the unit normal vector field n pointing away from z-axis

II) Problem 2 : (12 points)

Consider the parametrized surface S :

Where and . Find the area of the portion of the surface S that lies inside the unit ball

III) Problem 3 (12 points)

Use a line integral to find the mass of a thin wire running along the parabola (P) : from if the density of the wire at any point is equal to x numerically

IV) Problem 4 : (24 points)

Let S be the cone , and let C be its base on the (xy)-plane. Find the counterclockwise circulation of the field :

Around C :

i) Directly

ii) Using Greens Theorem

iii) Using Stokes Theorem (in the outward direction)

V) Problem 5 : (30 points)

We are the given the following 2 vector fields :

Let (C) be the parabola of equation : and the segment between the points

i) parametrize both parts of (C) and compute the line integral where F is the vector field

ii) apply greens theorem where G is the vector field

iii) one of the 2 vector fields are conservative , find it and compute its potential function

iv) show that for any path (C ‘ ) that G has a positive outward flux

VII) Problem 6 : (6 points)

Let be time dependent electric and magnetic fields. Let S be a smooth surface with simple closed boundary. We define :

Maxwell equation suggests that ∇×E =

where the curl is taken with respect to only the components (x,y,z). Faraday law states the voltage around C is equal the negative rate of change of magnetic ﬂux through S. Show that Faraday’s law follows from the Maxwell equation mentioned above